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The calculation of bulk strain in oblique and inclined balanced sections

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Abstract—The calculation of bulk strain from balanced sections is a technique commonly used in the study of orogenic belts. The validity of a balanced section is often enhanced by controls such as seismic and/or borehole data. The construction of a conventional vertical section perpendicular to orogenic strike (i.e. in the transport direction) may not allow control data to be directly incorporated into the section. The control data can be projected into the plane of the section or, alternatively, the section may be constructed in an orientation such that it contains the available control data. In the case of a vertical section constructed at an oblique angle to the transport direction the bulk strain value obtained is a minimum and can be corrected to the true value of bulk strain in the perpendicular, vertical section by a simple trigonometric calculation. If the section is perpendicular to strike but inclined at an angle to the vertical, the correction of the bulk strain values is mathematically unique to the section. This problem can be overcome by using a digitizing table in conjunction with a simple computer program. The general case of a section both oblique to the strike direction and inclined to the vertical can be resolved by removing the effects of the inclination to produce a vertical, oblique section from which the bulk strain can be obtained and corrected. In realistic geological problems, the oblique section will be a more common problem than the inclined section. In both cases deviations of 10° and less introduce such small errors in the estimates of bulk strain that their effects can be ignored.

INTRODUCTION

THE CONSTRUCTION of balanced sections (Dahlstrom 1969) has become widespread following the review of balancing techniques by Hossack (1979). Balanced and restored sections are particularly valuable in that they can be used to calculate estimates of bulk shortening in orogenic belts. The usual practice is to construct vertical balanced sections which trend perpendicular to orogenic strike; I shall term these normal sections. Normal sections are usually constructed because the vertical is the standard section plane and the maximum displacement and transport direction are generally perpendicular to orogenic strike (Elliott 1976, Elliott & Johnson 1980, p. 72). However, a large number of equally admissible balanced sections can be constructed for a given limited amount of outcrop data. Geophysical and/or borehole data can reduce the number of admissible balanced sections, and one of these may be more compatible with the control data than the others. There are two methods of incorporating such control data into balanced sections.

(1) The data can be projected along strike or fold plunge into the normal section plane (e.g. Price 1981, figs. 1 and 2; Gwinn 1970, fig. 4; Jacobeen & Kanes 1974, figs. 5 and 6).

(2) The section can be constructed in an orientation that 'best fits' the available control data, thus minimizing the distance of projection (e.g. Bally *et al.* 1966, section E–E').

I think that the second method is preferable when control data would otherwise be projected over considerable distances. The projection of data into the normal section plane may be invalidated by the lateral impersistence of structures, a problem discussed by Kilby & Charlesworth (1980) in relation to down-plunge projections. An example is the tendency of tectonic windows, formed by doming above duplexes at lower levels, to die out along strike (Boyer 1976, Boyer & Elliott 1982, Elliott & Johnson 1980, p. 77). In a normal section plane beyond the window, the same amount of bulk shortening may be present, accommodated by other structures, which do not exist in the window. It is clearly not valid in such a case to project control data derived within the window into the normal section plane beyond it.

There are also other reasons for constructing balanced sections with orientations that are abnormal. The most common reason is the constraint of good outcrop; road and rail cuts, quarry faces, etc. often have an abnormal orientation (e.g. Cooper *et al.* 1982). Exposures of this sort are invaluable in obtaining accurate local estimates of bulk strain which subsequently can be integrated to estimate an orogenic bulk strain (Hossack 1978). Other possibilities are: (1) coastal exposures not perpendicular to orogenic strike (e.g. the Hercynian orogen in SW Ireland); (2) non-vertical slopes in mountain belts that display structures (e.g. the Rockies, Thompson 1981) and (3) profiles constructed perpendicular to the axes of obliquely trending and/or plunging folds (Elliott & Johnson 1980, Kilby & Charlesworth 1980).

Hossack (1979) noted that given the assumptions of balancing methods the estimates of bulk strains from balanced sections will be minima. If an abnormal balanced section is constructed the bulk strain calculated will differ from the true value obtained in a normal section. To obtain the true value of bulk strain a correction must be applied to the bulk strain from the abnormal section. In this paper I wish to show that this can be done quickly and conveniently thus avoiding the need to project control data into normal sections. I will consider

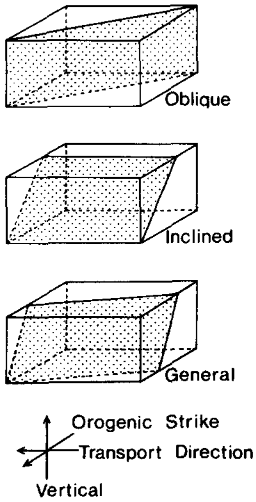

Fig. 1. The three types of abnormally oriented sections. The front faces of the blocks are all normal sections; vertical and parallel to the transport direction.

three types of abnormal section, the oblique case, the inclined case and the oblique and inclined or general case (Fig. 1).

OBLIQUE SECTIONS

An oblique section, unlike a normal section, does not generally contain the transport direction and will change its orientation during deformation. Oblique sections are the commonest type of abnormal section. In Fig. 2, EHOL is the undeformed oblique section which is at angle α to the undeformed normal section, EIPL. ACNL is the deformed oblique section which now makes an angle α' with the deformed normal section, ADQL. Section ACNL represents a deformed oblique balanced section whose angle of obliquity, α', is known. The deformed length of this section l_1' (EG) can be measured directly and the original length lo' (EH) can be calculated by summing deformed bed lengths in the section. The value of bulk shortening in the oblique section, e', may then be calculated. The parameters lo and l_1 in the normal section can be calculated from the following equations:

$$l_1 = l_1' \cdot \cos \alpha', \quad (1)$$

$$\sin \alpha = \frac{\tan \alpha' (\cos \alpha' \cdot l_1')}{lo'}, \quad (2)$$

$$lo = lo' \cdot \cos \alpha. \quad (3)$$

The value of bulk shortening in the equivalent normal section, e, can then be calculated. The result of simulating the relations between e and e' for various values of α' (Fig. 3) can be used to correct values of bulk shortening from oblique sections. If the value of α' is less than 10° no significant correction results, high α' values in contrast need large corrections especially at low values of bulk strain (Fig. 3). The gradients of the α' curves mean that lower values of bulk strain will be subject to a larger

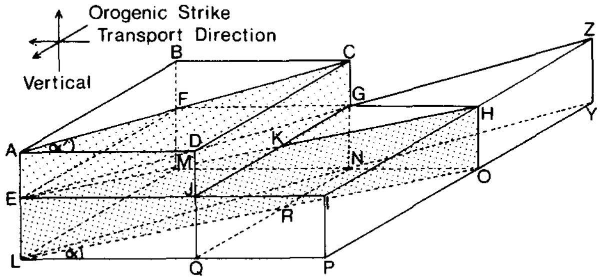

Fig. 2. The trigonometric relationships of the deformed and undeformed oblique sections. The deformed oblique section, ACNL, is shown by light stipple, the undeformed oblique section, EHOL, by heavy stipple. Note the change in the angle of obliquity from α to α' in the deformed section. The normal section is the front face of the block, ADJIPL.

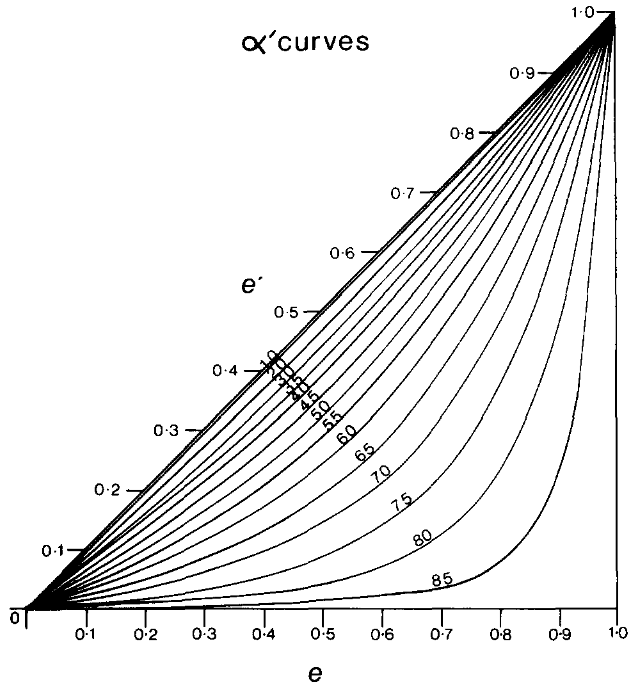

Fig. 3. α' correction curves derived for oblique sections. The bulk strain in the oblique section is e', that in the normal section is e. The numbers on the α' curves refer to the angle of obliquity of the section measured from the transport direction.

percentage correction than higher values. Price (1981, p. 436) suggests that tectonic displacements calculated in oblique sections vary with the cosine of the angle of obliquity; this is incorrect due to the change in angle from the deformed to the undeformed section.

The method described makes a number of assumptions. (1) The movement direction is perpendicular to orogenic strike, a factor discussed earlier. (2) The normal section displays plane strain, a problem discussed by Hossack (1979) who provided a method for correcting along-strike elongation that can be readily adapted for along-strike contraction. (3) The reference beds were originally parallel to the Z direction of the finite bulk strain ellipsoid. If this condition is not met a minimum of bulk strain will result.

The problem of strain inhomogeneity can be ignored as the calculation is for a bulk two-dimensional strain recorded by a deformed section. Whether the rocks deform by a homogeneous penetrative strain or by high strains on discrete surfaces separating essentially unstrained rock, the bulk strain calculated in the deformed section is the same. Material has moved through the section plane in an oblique section and it can be argued that this will invalidate bulk strain estimates where lateral ramps are significant. However, a similar problem exists with normal sections when oblique ramps are present. Normal sections are also problematic when lateral ramps lie within the section plane, as this will produce an incorrect bulk strain and be undetectable. An oblique section in contrast will transect lateral ramps allowing for their effects in the bulk strain estimate.

If the method of excess area (Hossack 1979) is used to calculate bulk strain then no correction need be applied. The excess area illustrated in Fig. 2, that is ACGE is divided by the original stratigraphic thickness, EL, yielding EZ–EG (the difference between original and deformed section length); EZ is not equal to *lo*′(EH). If EZ is constructed back in the same direction as the deformed oblique section (Fig. 2) the shortening calculated, e'', must be equal to the true shortening, e, by similar triangles,

$$e = e'' = \text{EG–EZ/EZ} = \text{EJ–EI/EI}. \qquad (4)$$

INCLINED SECTIONS

An inclined balanced section (Fig. 1b) is analogous to the outcrop of a plunging fold in a horizontal plane. The normal section is thus comparable to the fold profile plane and can be constructed from the inclined section by that method. Most lines in the inclined section will be longer than in the normal section. The percentage increase in line-length will depend on the dip of the inclined section and the orientation of the line within that section, conventionally expressed as pitch. The effects of an inclined section on line length are illustrated in Fig. 4. The line ab has increased by 90% in the inclined section (a′b′), whereas the line cd with its lower angle of pitch has only increased by 43% (c′d′). The areas beneath the lines in the section will be similarly affected. This precludes the derivation of simple correction curves as in the oblique case. The effect of an inclined section on bulk strain estimates is to yield a maximum value in contrast to that of the oblique section which yields a minimum.

If an inclined balanced section is constructed (GBDJ in Fig. 5) the intersection of a marker bed, FCIJ, with the inclined section can be measured directly, this is CJ (*lo*′ is the apparent original bed length). The pitch of CJ in the section (β) and the dip of the section (θ) are known and FJ (*lo*), the correct original bed length in the normal section can be calculated by:

$$\tan \gamma = \cos \beta \cdot lo'/\sin \theta(\sin \beta \cdot lo'), \qquad (5)$$

$$lo = \cos \beta \cdot lo'/\sin \gamma. \qquad (6)$$

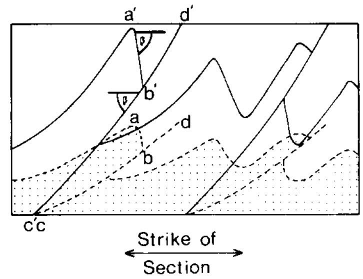

Fig. 4. The effect of an inclined section is to increase line length and hence section area. This increase depends on the dip of the section plane, θ, and the pitch of a line in the section plane, β. The section illustrated dips at 60°. The equivalent normal section of the structure is shown with a strippled ornament.

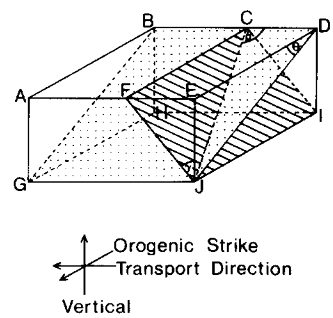

Fig. 5. The inclined section GBDJ is shown stippled; the intersecting bed FCIJ is ruled. The parameters CJ, β and θ can be measured from the section. The front face of the block is the normal section.

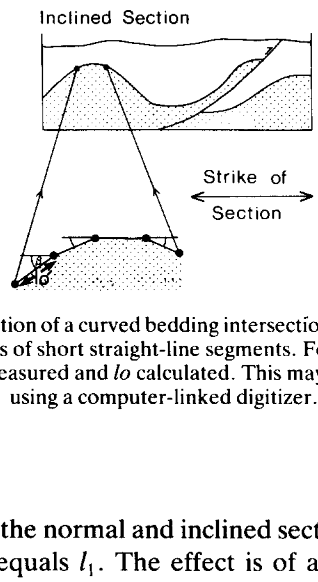

Fig. 6. The reduction of a curved bedding intersection from an inclined section to a series of short straight-line segments. For each segment β and *lo*′ can be measured and *lo* calculated. This may be automated by using a computer-linked digitizer.

The strike of the normal and inclined sections is parallel and thus l_1' equals l_1. The effect is of a mathematical projection of the lines from the inclined section plane into the normal section plane. The correct bulk strain may now be calculated.

In a real situation, such a simple geometric arrangement of beds is unlikely. This problem is overcome by dividing each bedding plane intersection in the inclined section into a number of small straight-line segments and performing a separate calculation for each (Fig. 6). If the method of excess area is used the area beneath each straight-line segment can be calculated. To correct a section by hand would be tedious and prone to error; simple digitizing systems make the process feasible.

In conjunction with a computer program to perform the calculations on the output data a digitizer would effectively automate the system. The assumptions made by this method are the same as for the oblique section case. In the case of inclined sections, however, the transport direction lies within the section plane removing some of the problems discussed in relation to oblique sections.

GENERAL SECTIONS

The general section is both inclined from the vertical and oblique to the transport direction (Fig. 1). To calculate bulk strain from such a section no simple correction can be applied. The effect of inclination must be removed first and the curve correction (Fig. 3) applied to the bulk strain calculated in the resultant vertical oblique section. The reverse sequence would yield an incorrect result because the α' curves (Fig. 3) apply only to vertical sections and as shown above cannot be derived for inclined sections. In removing the effects of inclination from the section it is merely being mathematically pivoted about the transport direction which parallels section strike.

APPLICATIONS

There are few published examples of balanced sections with abnormal orientations; those available are used below to illustrate the application of the correction techniques discussed.

The Dundonnell structure

The Dundonnell structure (Elliott & Johnson 1980) is situated near Ullapool in the Moine thrust belt of NW Scotland. A balanced and partially restored section was constructed by Elliott & Johnson (1980, p. 90, figs. 23 and 24) perpendicular to the trend of the Dundonnell anticline. Their construction was to illustrate the development of the anticline by the stacking of horses above an oblique ramp. The section is at an angle of 57° to the transport direction, that is $\alpha' = 57°$. The shortening of horses III, IV & V in the section (Elliott & Johnson 1980, fig. 24) can be calculated as 55% (longitudinal strain). On correction for the obliquity of the section using the α' curves (Fig. 3) the recalculated shortening in the transport direction is 74%. Elliott & Johnson (1980) outline a similar type of correction in terms of absolute displacements rather than bulk strain.

The Henaux Basse Normandie duplex

This well-exposed duplex in the Hercynian belt of NW France is described by Cooper *et al.* (1982, 1983). The duplex is exposed in the NW quarry wall and an adjacent railway cutting both of which dip at 80° and trend at an angle of 20° to the movement direction of the thrusts. This therefore provides an example of a general section. A line length balance of the duplex yields a bulk shortening of 19.7%. On correcting this value for the inclination of the section (10° from the vertical) the shortening value is reduced to 19.6%. Thus, as with oblique sections, an inclination of 10° or less from the vertical produces no significant change in the bulk strain estimate obtained. The α' curves (Fig. 3) can now be used to correct the bulk strain for the obliquity of the section. The α' value is 20°, producing a corrected bulk shortening of 21.5% from the vertical oblique section. This represents a 10% increase on the original bulk strain estimate.

CONCLUSIONS

This paper demonstrates the ease with which values of bulk strain calculated from sections constructed with abnormal orientations can be corrected to yield the bulk strain in the normal section plane. I, therefore, suggest that where outcrop distribution and/or control data dictate the construction of an abnormal section, it should be produced and the bulk strain estimates corrected using the procedures outlined. This avoids the assumption of the lateral persistence of structures that is critical to the projection of data from its actual location into a normal section plane. The subsequent correction of bulk strain thus allows abnormal sections to be constructed which are controlled by observed rather than inferred data, and therefore they are more geologically valid interpretations.

A deviation from the normal section of less than 10° inclination to the vertical and/or less than 10° obliquity will not significantly alter the bulk strain estimate, and no correction need be applied in such cases. In certain general sections the effects of inclination and obliquity will cancel one another: the conditions under which this will occur is however unique to that section.

The methods of correction derived are mathematically simple. More complex methods could be developed, these would however rest on further assumptions thus making their application more difficult. I have opted for simple methods to facilitate quick and more convenient correction of bulk strains from abnormally oriented sections.

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REFERENCES

Bally, A. W., Gordy, P. L. & Stewart, G. A. 1966. Structure, seismic data, and orogenic evolution of southern Canadian Rocky Mountains. *Bull. Can. Petrol. Geol.* **14**, 337–381.

Boyer, S. E. 1976. Formation of the Grandfather Mountain window, North Carolina, by duplex thrusting. *Abst. geol. Soc. Am. with Prog.* **8**, 788–789.

Boyer, S. E. & Elliott, D. 1982. The geometry of thrust systems. *Bull. Am. Ass. Petrol. Geol.* **66**, 1196–1230.

Cooper, M. A., Garton, M. R. & Hossack, J. R. 1982. Strain variation in the Henaux Basse Normandie duplex, northern France. *Tectonophysics* **88**, 321–323.

Cooper, M. A., Garton, M. R. & Hossack, J. R. 1983. The origin of the Basse Normandie Duplex, Boulonnais, France. *J. Struct. Geol.* **5**, 000–000.

Dahlstrom, C. D. A. 1969. Balanced cross sections. *Can. J. Earth Sci.* **6**, 743–757.

Elliott, D. 1976. The energy balance and deformation mechanisms of thrust sheets. *Phil. Trans. R. Soc.* **A283**, 289–312.

Elliott, D. & Johnson, M. R. W. 1980. Structural evolution in the northern part of the Moine thrust belt, N. W. Scotland. *Trans. R. Soc. Edinb., Earth Sci.* **71**, 69–96.

Gwinn, V. E. 1970. Kinematic patterns of lateral shortening. Valley and Ridge and Great Valley provinces, Central Appalachians, south-central Pennsylvania. In *Studies of Appalachian Geology—Central and Southern* (edited by Fisher, G. W. *et al.*). Wiley, New York, 127–146.

Hossack, J. R. 1978. The correction of stratigraphic sections for tectonic finite strain in the Bygdin area, Norway. *J. geol. Soc. Lond.* **135**, 229–242.

Hossack, J. R. 1979. The use of balanced cross-sections in the calculation of orogenic contraction; a review. *J. geol. Soc. Lond.* **136**, 705–711.

Jacobeen, F. & Kanes, W. H. 1974. Structure of Broadtop synclinorium and its implications for Appalachian structural style. *Bull. Am. Ass. Petrol. Geol.* **58**, 362–375.

Kilby, W. E. & Charlesworth, H. A. K. 1980. Computerized downplunge projection and the analysis of low-angle thrust-faults in the Rocky Mountain foothills of Alberta Canada. *Tectonophysics* **66**, 287–299.

Price, R. A. 1981. The Cordilleran foreland thrust and fold belt in the southern Canadian Rocky Mountains. In: *Thrust and Nappe Tectonics* (edited by McClay, K. R. & Price, N. J.). *Spec. Publs geol. Soc. Lond.* **9**, 427–448.

Thompson, R. J. 1981. The nature and significance of large 'blind' thrusts within the northern Rocky mountains of Canada. In: *Thrust and Nappe Tectonics* (edited by McClay, K. R. & Price, N. J.). *Spec. Publs geol. Soc. Lond.* **9**, 449–462.